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Ship Hydromechanics Department

A MATHEMATICAL MODEL FOR SURFACE SHIP MANEUVERING

by

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DTRC/SHD-1350-01 A Mathematical Model for Surface Ship Maneuvering

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ABSTRACT

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INTRODUCTION

This report documents the current state of a time domain simulation for the maneuvering of surface ships, including the effect of unsteady wind forces (wind gusts), slowly-varying wave drift forces in long- or short-crested seas, and forces due to a towed body. This model is suitable for trackkeeping studies as well as traditional maneuverability studies. It has evolved at the David Taylor Research Center (DTRC) over a period of more than two decades from the original three-degree-of-freedom model of Strom-Tejsen[1], based on the mathematical model of Abkowitz[2]. Many people have worked on it, or influenced its development.

Motter[1] added a fourth degree of freedom (roll) and the propeller and rudder model of Smit and Chislett [3]. McCreight and Motter[1] carried out a major rewrite of this program as the first stage of an effort to develop a full maneuvering in waves model. In addition to structuring the program, a steady wind model was added, coefficients were added to be valid over an extended speed range instead of merely accounting for "small" deviations from the initial conditions, improved roll damping. Provision was made for defining an arbitrary sequence of rudder and speed commands. A nonlinear initialization scheme was developed to handle the large excursions which can be caused by strong winds. Finally, the previously used Euler integration was replaced by an adaptive Runge-Kutta which allowed the use of large step sizes

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[1]Published in reports with limited distribution.

while maintaining accuracy. This was particularly important due to the sensitivity of the model in roll. In addition, a detailed model of steam turbine, gas turbine, and diesel engines was incorporated by Propulsion Dynamics, Inc. (PDI)[4]. PDI also produced a user's manual for this program. This maneuvering program was known as WAVMAN44 for historical reasons, even though it has no wave effects. WAVMAN44 has been used in many maneuvering studies, and has been the base for two further developments.

The first and most extensive of the developments is the originally planned maneuvering in waves simulation, which added two additional degrees of freedom (heave and pitch) to produce a full six-degrees-of-freedom and added first order wave excitations and reaction forces, including memory effects (which result in the frequency-dependent added mass and damping effects in the frequency domain). Regular wave, and long- and short-crested random seas models were included. The results of this effort are described by McCreight[5]. The resultant computer program has been referred to as WAVMAN.

The other branch of development for WAVMAN44 was by Waters, Hickok, Turner, Hart, and Bochinski of DTRC. They implemented a series of efforts which are either presented in reports of limited distribution or are undocumented. Their calm water hydrodynamic model was believed to be more appropriate for SWATH ships with dihedral fins forward of the propellers, and included the addition of fixed fin effects (together with heave and pitch for examining the sinkage and trim changes due to fins), an unsteady wind model, a mean wave drift force model, forces due to a towed body, and a simple autopilot. Equations for calm water forces for heave and pitch were added. This program has been referred to as TOWMAN87.

The present version of the program, MPSS (Maneuvering Program for Surface Ships), is applicable to both monohull and SWATH designs. While TOWMAN87 served as the starting point, numerous features from WAVMAN have been incorporated in MPSS. MPSS also has components and characteristics not included in any of the predecessor programs. A list of the major differences and similarities between MPSS and the other programs follows.

(1) MPSS is a six-degree-of-freedom model as is WAVMAN. While TOWMAN87 includes some heave and pitch calm water hydrodynamic and hydrostatic terms, it does not include the necessary inertial terms and transformations between coordinate systems required for a consistent six degrees of freedom model.

(2) The terms used in the calm water hydrodynamic model in MPSS were selected on the basis of physical considerations. The terms in the model were examined for the symmetry or asymmetry, resulting in changes in the equations. Fourteen terms in the TOWMAN87 model were eliminated and 30 terms were added on this basis. The model is most similar to that of TOWMAN87. For flexibility, the capability to utilize the model implemented in WAVMAN (and WAVMAN44) has been retained.

(3) MPSS incorporates an explicit model to account for the effects of the propeller-rudder interaction on the forces generated by the rudder similar to that developed by Norrbin[6], Abkowitz[7], and others. This is extremely important in modeling the motions of vessels with rudder(s) aft of the propeller(s). This configuration, which occurs in most monohulls as well as SWATHs with overhanging struts, results in the propeller wake flowing over the rudder. This model is also applicable to ships with rudders forward of the propeller.

(4) A linear interpolation method for accounting for the ship resistance was adopted in MPSS.

(5) Approximations for the wind forces from WAVMAN are included in MPSS. The unsteady wind model and the forces due to a towed body from TOWMAN87 have been retained.

(6) A slowly-varying wave drift force model has been developed and implemented in MPSS.

(7) MPSS has features not present in either WAVMAN or TOWMAN87 which do not affect the modeling. These include a new data input scheme which is more convenient and verifies variable names.

The changes in the surge force model in items (2), (3), and (4) above together contribute to a greatly improved speed loss model. Another effect of these changes is that only one set of coefficients is required, which is valid for all speeds, instead of separate sets at each of the speeds which is sometimes done.

The resultant mathematical model which is implemented in MPSS is described in this report. As noted above, documentation of several intermediate stages of the developments of this model has been in limited distribution reports. The distribution was limited due to the inclusion of hydrodynamic coefficients and predicted maneuvering performance for specific designs in those reports. For this reason, such detailed data has been omitted from this report.

EQUATIONS OF MOTION

Newton's laws of motion form the basis of calculating the acceleration of the ship, considered to be a rigid body subject to external forces due to the propulsion system, rudder, fins, the relative motion of the water due to the motion of the ship, and environmental forces due to wind and waves. These accelerations are then integrated twice to obtain the time evolving position and orientation of the ship as it maneuvers.

COORDINATE SYSTEMS

In developing the equations of motion, several related coordinate systems are used. These are presented here in the order of the natural succession from one to the next.

These are:

1) The *inertial* (x_0, y_0, z_0) system which is right handed, with z_0 positive downwards, and is fixed in space.
2) The *upright* (x_u, y_u, z_u) coordinate system is parallel to the inertial coordinate system with its origin on the free surface, coinciding with amidships and moving with the ship.
3) The *yawed* (x', y', z') coordinate system is obtained by rotation of the upright coordinate system through the angle ψ about the z_u axis.
4) The *pitched* (x", y", z") coordinate system is obtained by rotating the yawed coordinate system through the angle θ about the y' axis.
5) The *body* or *maneuvering* (x, y, z) coordinate system is obtained by rotating the pitched coordinate system through the angle ϕ about the x" axis. This is fixed and is the conventional SNAME (Society of Naval Architects and Marine Engineers) coordinate system[8].

DYNAMICS

The equations of motion assuming: (a) transverse symmetry, (b) the principal axes of inertia parallel to the chosen axis system, $I_y = I_z$, and (c) the center of gravity located at (x_G, 0, z_G), are, in standard nomenclature[8]:

$$X = m[\dot{u} + qw - rv - x_G(q^2 + r^2) + z_G(pr + \dot{q})] \tag{1}$$

$$Y = m[\dot{v} + ru - pw + z_G(qr - \dot{p}) + x_G(pq + \dot{r})] \tag{2}$$

$$Z = m[\dot{w} + pv - qu - z_G(p^2 + q^2) + x_G(pr - \dot{q})] \tag{3}$$

$$K = I_x\dot{p} + (I_z - I_y)qr - mz_G(\dot{v} + ru - pw) - (I_{xz} + mx_Gz_G)(pq + \dot{r}) \tag{4}$$

$$M = I_y\dot{q} + (I_x - I_z)\,p\,r + m\,z_G\,(\dot{u} + q\,w - r\,v) - m\,x_G\,(\dot{w} + p\,v - q\,u)$$

$$+ (\,I_{xz} + m\,x_G\,z_G\,)\;(\,p^2 - r^2\,) \qquad (5)$$

$$N = I_z\dot{r} - (\,I_x - I_y\,)\,pq + m x_G(\,\dot{v} + r\,u - p\,w) + (\,I_{xz} + m\,x_G\,z_G\,)(\,q\,r - \dot{p}) \qquad (6)$$

In the above equations, u, v, and w are the surge, sway and heave velocities, and p, q, and r are the roll, pitch, and yaw velocities, measured in the body coordinate system. X, Y, Z, K, M, and N are the total external surge, sway, heave, roll, pitch, and yaw forces and moments, respectively. m is the ship mass, I_x, I_y, and I_z are the mass moments of inertial about the x, y, and z axis, respectively. I_{xz} is the product of inertia with respect to the x and z axis.

The following equations relate the time rate of change of position and orientation to the velocities in maneuvering coordinates:

$$\dot{x}_0 = u\cos\theta\cos\psi + v\,(\,\sin\phi\sin\theta\cos\psi\ - \cos\phi\sin\psi\,) + w\,(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) \quad (7)$$

$$\dot{y}_0 = u\cos\theta\sin\psi + v\,(\,\sin\phi\sin\theta\sin\psi\ + \cos\phi\cos\psi\,) + w\,(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) \quad (8)$$

$$\dot{z}_0 = -u\sin\theta + v\sin\phi\cos\theta + w\cos\phi\cos\theta \qquad (9)$$

$$\dot{\phi} = p + (q\sin\phi + r\cos\phi)\tan\theta \qquad (10)$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \qquad (11)$$

$$\dot{\psi} = (q\sin\phi + r\cos\phi)\,/\,\cos\theta \qquad (12)$$

When integrated, these equations yield (x_0, y_0, z_0), the position of the ship origin in the inertial coordinate system, and the Euler angles ϕ, θ, and ψ which are commonly known as the roll, pitch and yaw angles.

The transformation of a vector from the yawed coordinate system to the maneuvering coordinate system, assuming x = z = 0, is given by

$$F_{MAN} = \begin{vmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\phi\sin\theta & \cos\phi & \sin\phi\cos\theta \\ \cos\phi\sin\theta & -\sin\phi & \cos\phi\cos\theta \end{vmatrix} F_{YAW} \qquad (13)$$

where F_{MAN} and F_{YAW} are the vectors in the maneuvering and yawed coordinate systems, respectively. This is required to transform force and moment vectors computed in the yawed coordinate system into the maneuvering coordinate system.

EXTERNAL FORCES AND MOMENTS

Combining the external forces and moments into a vector:

$$F = \{X, Y, Z, K, M, N\}^T$$

where T indicates the transpose.

We can separate the forces into components according to the cause of each force and moment:

$$F = F_{HS} + F_{CW} + F_{WIND} + F_{WAVE} + F_{TOW} \quad (14)$$

The subscript HS indicates hydrostatic force, and all six components must be considered due to the changing orientation of the ship axis system. The subscript CW indicates the maneuvering force affecting the hull in calm water. These include the bare hull, propeller, rudder, and fin, if any. The next two terms model external forces due to environmental disturbances. In the present work, unsteady wind forces and slowly-varying wave drift forces are modelled. The final term models the forces due to a towed body.

All force and moment components in the equation above are each calculated in a coordinate system which is convenient for that component, then transformed into the maneuvering coordinate system, if necessary, and summed to obtain the total forces and moments acting on the ship. These models are described in detail below.

HYDROSTATIC FORCES

The hydrostatic forces and moments which represent the difference between the gravitational forces on the ship and the integral of the hydrostatic pressure acting on the hull are calculated using a linear theory in the yawed coordinate system. In this system the non-zero components of the hydrostatic forces and moments are taken as:

$$Z_{HS} = -z_0 C_{33} - \theta C_{35}$$

$$K_{HS} = -\phi C_{44}$$

$$M_{HS} = -z_0 C_{35} - \theta C_{55}$$

where z_0 is the vertical displacement of the seakeeping origin from its rest position, and ϕ and θ are the (finite amplitude) roll and pitch angles, which for small angles reduce to the corresponding linear seakeeping theory angles. The C_{ij} terms are the conventional linear hydrostatic coefficients defined by, for example, Salvesen, Tuck, and Faltinsen[9]. The resulting forces and moments are then transformed into the body coordinate system.

CALM WATER HYDRODYNAMIC FORCES

The calm water terms include the traditional maneuvering model and have evolved over many years. The history and development of these terms was briefly outlined in the introduction. The calm water forces and moments are represented as the sum of the product of coefficients and state variables. In the computer program, all terms are non-dimensionalized using the SNAME system[8]. Following solution of the system of equations of motion, velocities are dimensionalized. For simplicity, the equations presented here will be given in the dimensional form. These forces are calculated in the maneuvering coordinate system.

The final expression for surge, sway, and heave forces and roll, pitch, and yaw moments follow.

$$
\begin{aligned}
X_{CW} = {} & X_{\dot{u}}\dot{u} + X_0 + (X_{vv} + X_{vvF}\,F)\,v^2 + (X_{rr} + X_{rrF}\,F)\,r^2 + (X_{vr} + X_{vvrr}\,v\,r)\,v\,r \\
& + (X_{v\phi}\,v + X_{r\phi}\,r)\,\phi + (X_{\phi\phi} + X_{vv\phi\phi}\,v^2 + X_{rr\phi\phi}\,r^2 + X_{vr\phi\phi}\,v\,r)\,\phi^2 \\
& + X_{\delta\delta}\,\delta^2 + \sum_{k=1}^{NF} X_{F_k}\delta_{F_k} \\
& + (X_{ccee} + X_{ccee\phi\phi}\,\phi^2)\,c^2e^2 + X_{e\phi}\,e\,\phi + X_{RP}
\end{aligned}
$$

$$
\begin{aligned}
Y_{CW} = {} & (Y_{\dot{v}} + Y_{\dot{v}u}\,u + Y_{\dot{v}uu}\,u^2)\,\dot{v} + Y_{\dot{w}}\,\dot{w} + Y_{\dot{p}}\,\dot{p} + Y_{\dot{q}}\,\dot{q} + (Y_{\dot{r}} + Y_{\dot{r}u}\,u + Y_{\dot{r}uu}\,u^2)\,\dot{r} \\
& + (Y_v + Y_{vF}\,F + Y_{vFF}\,F^2)\,v + Y_p\,p + (Y_r + Y_{rF}\,F + Y_{rFF}\,F^2)\,r \\
& + Y_{v|v|}\,v\,|v| + Y_{v|r|}\,v\,|r| + Y_{r|r|}\,r\,|r| + Y_{r|v|}\,r\,|v| \\
& + (Y_\phi + Y_{\phi F}\,F + Y_{\phi FF}\,F^2)\,\phi \\
& + (Y_{vv\phi}\,v + Y_{v\phi\phi}\,\phi)\,v\,\phi + Y_{vr\phi}\,v\,r\,\phi + (Y_{rr\phi}\,r + Y_{r\phi\phi}\,\phi)\,r\,\phi \\
& + (Y_\delta + Y_{\delta\delta\delta}\,\delta^2 + Y_{\delta FF}\,F^2)\,\delta + Y_{\delta|r|}\,\delta\,|r| + \sum_{k=1}^{NF} Y_{F_k}\delta_{F_k} \\
& + (Y_{cce} + Y_{cceee}\,e^2 + Y_{cce\phi\phi}\,\phi^2)\,c^2 e
\end{aligned}
$$

$$
\begin{aligned}
Z_{CW} = {} & Z_{\dot{v}}\,\dot{v} + Z_{\dot{w}}\,\dot{w} + Z_{\dot{p}}\,\dot{p} + Z_{\dot{q}}\,\dot{q} + Z_{\dot{r}}\,\dot{r} + Z_u\,u + Z_v\,v + Z_w\,w + Z_q\,q + Z_r\,r \\
& + Z_z\,z_0 + Z_\theta\,\theta + Z_\delta\,\delta + \sum_{k=1}^{NF} Z_{F_k}\delta_{F_k}
\end{aligned}
$$

$$K_{CW} = (K_{\dot{v}} + K_{\dot{v}u}\,u + K_{\dot{v}uu}\,u^2)\,\dot{v} + K_{\dot{w}}\,\dot{w} + K_{\dot{p}}\,\dot{p} + K_{\dot{q}}\,\dot{q} + K_{\dot{r}}\,\dot{r}$$

$$+ (K_v + K_{vF}\,F + K_{vFF}\,F^2)\,v + K_p\,p + (K_r + K_{rF}\,F + K_{rFF}\,F^2)\,r$$

$$+ K_{v|v|}\,v\,|v| + K_{v|r|}\,v\,|r| + K_{r|r|}\,r\,|r| + K_{r|v|}\,r\,|v|$$

$$+ (K_\phi + K_{\phi F}\,F + K_{\phi FF}\,F^2)\,\phi$$

$$+ (K_{vv\phi}\,v + K_{v\phi\phi}\,\phi)\,v\,\phi + K_{vr\phi}\,v\,r\,\phi + (K_{rr\phi}\,r + K_{r\phi\phi}\,\phi)\,r\,\phi$$

$$+ (K_\delta + K_{\delta\delta\delta}\,\delta^2 + K_{\delta FF}\,F^2)\,\delta + \sum_{k=1}^{NF} K_{F_k}\delta_{F_k}$$

$$+ (K_{cce} + K_{cceee}\,e^2 + K_{cce\phi\phi}\,\phi^2)\,c^2 e$$

$$M_{CW} = M_{\dot{v}}\,\dot{v} + M_{\dot{w}}\,\dot{w} + M_{\dot{p}}\,\dot{p} + M_{\dot{q}}\,\dot{q} + M_{\dot{r}}\,\dot{r} + M_u\,u + M_v\,v + M_w\,w + M_q\,q + M_r\,r$$

$$+ M_z\,z_0 + M_\theta\,\theta + M_\delta\,\delta + \sum_{k=1}^{NF} M_{F_k}\delta_{F_k}$$

$$N_{CW} = (N_{\dot{v}} + N_{\dot{v}u}\,u + N_{\dot{v}uu}\,u^2)\,\dot{v} + N_{\dot{w}}\,\dot{w} + N_{\dot{p}}\,\dot{p} + N_{\dot{q}}\,\dot{q} + (N_{\dot{r}} + N_{\dot{r}u}\,u + N_{\dot{r}uu}\,u^2)\,\dot{r}$$

$$+ (N_v + N_{vF}\,F + N_{vFF}\,F^2)\,v + N_p\,p + (N_r + N_{rF}\,F + N_{rFF}\,F^2)\,r$$

$$+ N_{v|v|}\,v\,|v| + N_{v|r|}\,v\,|r| + N_{r|r|}\,r\,|r| + N_{r|v|}\,r\,|v|$$

$$+ (N_\phi + N_{\phi F}\,F + N_{\phi FF}\,F^2)\,\phi$$

$$+ (N_{vv\phi}\,v + N_{v\phi\phi}\,\phi)\,v\,\phi + N_{vr\phi}\,v\,r\,\phi + (N_{rr\phi}\,r + N_{r\phi\phi}\,\phi)\,r\,\phi$$

$$+ (N_\delta + N_{\delta\delta\delta}\,\delta^2 + N_{\delta FF}\,F^2)\,\delta + N_{\delta|r|}\,\delta\,|r| + \sum_{k=1}^{NF} N_{F_k}\delta_{F_k}$$

$$+ (N_{cce} + N_{cceee}\,e^2 + N_{cce\phi\phi}\,\phi^2)\,c^2 e$$

In the above F is the Froude number, defined by $\frac{U}{\sqrt{gL}}$ where $U = \sqrt{u^2 + v^2 + w^2}$ is the instantaneous forward speed of the ship and δ is the rudder angle.

N_F is the number of fins installed, δ_{F_k} is the angle of the k^{th} fin. The fin model is included to account for the effects of fins, particularly on SWATHs. As currently implemented, the fin angle is assumed to be a constant which is set at the beginning of the run; that is, no active control of fins is considered. The fins are assumed to be simple lifting surfaces mounted at

an arbitrary location and angle. The X force is proportional to the square of deflection, δ_{F_i}. It is assumed that X_F is minimum at $\delta_F = 0$.

The term X_{RP} and the terms with coefficients with c and e for subscripts can be used to represent the resistance and propulsion model. Details of the resistance and propulsion model are given in the following section.

The terms included in this hydrodynamic model were selected by considering the physics of the flow around the hull, propellers, and rudders, and the likely functional dependence of the resulting flow forces on the mean flow over the rudder, c, and the effective angle of attack, e. Moderate order polynomials in these variables and absolute values of v and r were considered. Terms were included in the model only if the resulting contribution would have the expected symmetry or anti-symmetry. That is, surge force is symmetric; sway force, yaw moment, and roll moment are asymmetric. Not all of the coefficients in the model are required for all ships. In the current model, some asymmetry, as for single-screw ships, may be excluded. In this case, appropriate terms may be added to the model.

Some terms included in several of the previous models mentioned in the introduction would not satisfy this criterion. Some other terms are ones for which data is not likely to be available. The appendix lists additional terms which have been utilized in previous models. These terms have been retained in the computer program to provide flexibility.

The numerical values may be obtained either from model experiments or by prediction. Smitt and Chislett[3], and reference 10 discuss model testing techniques. Acceleration terms ($Y_{\dot{v}}$, etc.) may be predicted with reasonable accuracy by potential theory. For other coefficients, predictions are usually not too reliable unless a similar hull is available. Prediction techniques for the coefficients are beyond the scope of the report here. For example, see reference 10 for information on the topic.

Resistance and Propulsion Model

The resistance and propulsion model calculates the net surge force X_{RP} due to the calm water resistance and the propellers. In this section, the current model for calculating X_{RP} as well as two older methods of calculating X_{RP} will be presented. The older methods are included for compatibility with older sets of coefficients.

The current method of calculating X_{RP} has several advantages over these older methods. First, flow due to the propellers is calculated and its effect on the rudder forces is modelled explicitly, making possible greater accuracy over widely varying operating conditions. Secondly, effects of speed on the non-dimensional coefficients should be eliminated, except at high Froude numbers, and thus a single set of coefficients should suffice for all speeds. In contrast, with the other models it is common to use separate sets of coefficients at different

speeds, which is necessary because in current practice the resistance and propulsion effects are imbedded in coefficients which do not explicitly depend upon Froude number (for resistance) and shaft speed (for thrust).

The net surge force due to calm water resistance and propulsion is given by:

$$X_{RP} = N_p \, x_{cprop} \, (1 - t) \, T - R_T$$

where N_p is the number of propellers, t is the thrust deduction, T is the open water propeller thrust per shaft, R_T is the calm water resistance, and x_{cprop} is a coefficient which corrects for possible inaccuracies in the calculated values of T, which is not necessarily well known. Resistance is obtained by linear interpolation in a table of resistance coefficients which are defined by:

$$C_R = \frac{R_T}{\frac{\rho}{2} U^2 L^2}$$

Propeller open water thrust and torque coefficients K_T and K_Q are represented in the form:

$$K_T = \frac{T}{\rho \, n^2 D^4} = c_{KT_0} + c_{KT_1} J_T + c_{KT_2} J_T^2 \qquad (15)$$

$$K_Q = \frac{Q}{\rho \, n^2 D^5} = c_{KQ_0} + c_{KQ_1} J_Q + K_{Q_2} J_Q^2 \qquad (16)$$

where J_T is the advance coefficient U_A / n D, n is the shaft speed, and D is the propeller diameter. The speed of advance U_A is given by U (1 - w_t) where w_t is the thrust wake fraction. J_Q is defined similarly for torque using w_q, the torque wake fraction. Given U, t, w, and a machinery model as discussed in the following section, equations (15) and (16) can be solved for n and T.

Finally, in this model two other quantities used in the force and moment model of the previous section are also calculated. The mean flow over a rudder is given by:

$$c = \sqrt{\frac{A_P}{A_R} [\, (1 - w_t) \, u + k \, U_{A\infty}]^2 + (\, 1 - \frac{A_P}{A_R}) \, (1 - w_t)^2 \, u^2}$$

where A_P is the area of the rudder directly behind the propeller and A_R is the area of the rudder, w_t is the wake fraction of the propeller jet velocity at infinity and $k = U_A / U_{A\infty}$ is a function of $\frac{(x_r - x_p)}{D/2}$ where x_r is the rudder location and x_p is the propeller location. k is derived theoretically in reference 11. Reference 7 contains a convenient plot of this function. $U_{A\infty}$ is

given by:

$$U_{A\infty} = -(1 - w_t)\, u + \sqrt{(1 - w_t)^2 u^2 + \frac{8K_T}{\pi}(n\, D)^2}$$

Finally, the effective angle of attack of the flow over the rudder is given by:

$$e = \delta - \tan^{-1}\left[\frac{v}{c} + \frac{r\, x_r}{c}\right].$$

This is the version of propeller-rudder interaction model proposed by Abkowitz[7] following Norrbin[6]. Use of c and e makes possible a more accurate calculation of the influence of propeller speed on the forces and moments on the ship.

The first of the two older models is:

$$X_{RP} = (1 - t)\, N_P\, T - R_T$$

where T is defined by equation (15). R_T for this model is given in pounds by:

$$R_T = \frac{550\, P_E}{U}$$

where P_E is the ship effective horsepower represented by a polynomial.

$$\frac{P_E}{N_P\, U^3} = c_{KP_0} + c_{KP_1} U + c_{KP_2} U^2 + c_{KP_3} U^3$$

The propeller thrust is calculated as in the first model.

The second of the two older models utilizes the simple expression:

$$X_{RP} = X_{\bar{u}}\bar{u} + X_{\bar{u}\bar{u}}\bar{u}^2 + X_{\bar{u}\bar{u}\bar{u}}\bar{u}^3$$

where $\bar{u} = u - U_0$ is the speed loss and U_0 is the initial forward speed of the ship. In this model, hull resistance, propeller, and (implicitly) machinery dynamics are all combined into a simple function of the speed loss.

Machinery Model

The machinery model governs the variation of propeller speed throughout the maneuver. There exist three simple models and two detailed models. For the simple machinery models, either speed, torque, or power is held constant. The detailed machinery models are for either a diesel engine or a turbine and include the effects of engine dynamics,

governors, torque limiters, and drive train dynamics. The detailed models were developed by Propulsion Dynamics, Inc.[4] and will not be included here.

WIND FORCES

The wind forces and moments, F_{WIND}, are b[illegible]ed on data, C_{WIND}, measured in a wind tunnel for a range of headings. The wind force and moment coefficients, C_{WIND}, are defined by non-dimensionalizing the measured forces by $\frac{\rho_A}{2} L^2 U^2$ and the moments by $\frac{\rho_A}{2} L^3 U^2$ where ρ_A is the mass density of air.

This non-dimensional data, C_{WIND}, are interpolated linearly in heading to obtain the values for the correct relative wind direction, $\psi_W - \psi$, where ψ_W is the wind heading:

$$\psi_{RW} = \tan^{-1}\left[\frac{U_W \sin(\psi_W - \psi) - v_I}{U_W \cos(\psi_W - \psi) - u_I}\right] \tag{17}$$

where u_I and v_I are the ship velocities in the inertial coordinate system. F_{WIND} is then calculated by dimensionalizing the resultant values, $C_{WIND}(\psi_{RW})$. The force components are dimensionalized by $\frac{\rho_A}{2} L^2 U_{RW}^2$ and the moment components by $\frac{\rho_A}{2} L^3 U_{RW}^2$ where

$$U_{RW} = \sqrt{[\, U_W \cos(\psi_W - \psi) - u_I \,]^2 + [\, U_W \sin(\psi_W - \psi) - v_I \,]^2} \tag{18}$$

The wind direction, ψ_W is measured relative to the x_0-axis, and is such that with the ship parallel to the x_0-axis, $\psi_W = 0°$ is a following wind, $\psi_W = 90°$ is a port beam wind, and $\psi_W = 180°$ is a head wind. In these equations, U_W is the wind speed which will be defined in the next section. These calculations are performed in the yawed coordinate system.

Unsteady Wind Model

An unsteady wind velocity model is incorporated using the Davenport[12] spectrum for variation of the longitudinal component of the wind. No lateral spectrum is used at present, so the direction of wind is assumed to be constant.

The unsteady wind velocity, $U_W(t)$, used in equations (17) and (18) is given by:

$$U_W(t) = \bar{U}_{WIND} + \sum_{j=1}^{N_\omega} \sqrt{2\, S_{WIND}(\omega_j)\, \Delta\omega_j}\; \cos(\omega_j t + \xi_j) \tag{19}$$

where $\bar{U}_{WIND}$ is the mean wind velocity in knots at an elevation of 10 meters above the water

surface and the Davenport wind spectrum density function $S_{WIND}(\omega_j)$ in m^2 / sec is given by :

$$S_{WIND}(\omega_j) = \frac{1.459 \times 10^5 \ \kappa \, \omega_j}{(1 + x^2)^{4/3}} \ ,$$

ω_j is the frequency in radians per second, ξ_j is the random phase, distributed uniformly over $(0,2\pi]$, κ is a drag coefficient taken as 0.005, and

$$x = 1.378 \times 10^5 \frac{\omega}{\bar{U}_{WIND}}$$

In the model, the spacing of the discrete ω_j is selected so that each component in the summation of equation(19) has the same amplitude; the frequencies are spaced more closely where the spectrum is larger.

Approximation for Wind Force Coefficients

If measured C_{WIND} data are not available, an alternative is the model applied previously by McCreight[5] for monohulls:

$$X_{WIND} = \frac{\rho_A}{2} U_{RW}^2 \ A_F \, C_{X_{w_{max}}} \sin\left(\frac{9}{7} (\mid \psi_{RW} \mid - \frac{\pi}{2}) \right) \qquad \mid \psi_{RW} \mid < \pi$$

$$Y_{WIND} = \frac{\rho_A}{2} U_{RW}^2 \ A_S \, C_{Y_{w_{max}}} \sin(\psi_{RW})$$

$$K_{WIND} = \frac{\rho_A}{2} U_{RW}^2 \ A_S \, H_S \, C_{K_{w_{max}}} \sin(\psi_{RW}) \cos^2\phi$$

$$N_{WIND} = \frac{\rho_A L}{2} U_{RW}^2 \ A_F \, C_{N_{w_{max}}} \sin(2\psi_{RW})$$

where A_F, A_S, H_S, ρ_A, ρ, and L are projected frontal and side above water areas, the vertical coordinate of the centroid of the projected side area of the ship, air mass density, water mass density, and ship length, respectively.

The values 0.70, 0.80, 1.30, 0.10 have been adopted for the wind force coefficients $C_{X_{w_{max}}}$, $C_{Y_{w_{max}}}$, $C_{K_{w_{max}}}$, and $C_{N_{w_{max}}}$, respectively. This approximation is based on experimental results of Aage[13] for a number of ships. The roll dependence is adapted from Sarchin and Goldberg[14].

WAVE FORCES

A seaway acting on a ship causes forces, some of which can be neglected in the maneuvering problem and some which must be accounted for. The incident seas can be represented using the model of St. Denis and Pierson[15], in which the sea surface $\zeta(x_0,y_0,t)$ is represented as the sum of sine waves of varying lengths and directions.

$$\zeta(x_0,y_0,t) = \mathrm{Re} \sum_{j=1}^{n} \sum_{k=1}^{m} e^{-iK_k(x_0 \cos\mu_j + y_0 \sin\mu_j) + i\omega_k t + i\varepsilon_{jk}} \sqrt{S_\zeta(\omega_k,\psi_j)\, \Delta\mu_j\, \Delta\omega_k}$$

for short-crested seas, where K_k is the wave number $\frac{\omega_k^2}{g}$ of the kth frequency ω_k of m frequencies, μ_j is the jth of n directions, ε_{jk} is the random phase of the wave component of frequency ω_k and direction μ_j, uniformly distributed over $(0, 2\pi]$ and S_ζ is the Bretschneider[16] wave spectrum with cosine-squared spreading given by

$$S_\zeta(\omega_k,\psi_j) = \frac{2}{\pi}\cos^2(\mu - \mu_c)\ S_\zeta(\omega_k) \qquad 0 < |\mu - \mu_c| \le \frac{\pi}{2}$$

$$= 0 \qquad \frac{\pi}{2} < |\mu - \mu_c| \le \pi$$

and

$$S_\zeta(\omega_k) = 487.1\, (\zeta_w)^2_{1/3}\, T_0^{-4}\, \omega^{-5}\, e^{1944.5\, T_0^{-4}\, \omega^{-4}}$$

where $(\zeta_w)^2_{1/3}$ is the significant wave height, T_0 is the modal, or peak energy, period and μ_c is the (predominant) wave direction. The wave direction μ is measured relative to the x_0 axis, and is such that with the ship parallel to the x_0-axis, $\mu = 0°$ is a following wave, $\mu = 90°$ is a port beam wave, and $\mu = 180°$ is a head wave. This convention is the same as for the wind heading.

For long-crested waves the appropriate expression is:

$$\zeta(x,y,t) = \mathrm{Re} \sum_{k=1}^{m} e^{-iK_k(x_0 \cos\mu + y_0(t) \sin\mu_j) + i\omega_k t + i\varepsilon_k} \sqrt{2S_\zeta(\omega_k)\, \Delta\omega_k}$$

where μ is the wave direction and ε_k is the random phase uniformly distributed over $(0,2\pi]$.

Ogilvie[17] presents an excellent survey of the resulting forces and methods of predicting them. Here we just give a brief summary of these forces and discuss the assumptions

and approximations used in the program for calculating the slowly-varying wave drift force components, as well as indicate why the other terms were omitted.

For purposes of explanation, we now consider two plane waves of different lengths propagating in the same direction

$$\zeta(t) = A_1 \cos\omega_1 t + A_2 \cos\omega_2 t$$

Assuming a perturbation expansion in the wave amplitudes A_1 and A_2 and ignoring terms greater than quadratic, the forces are of the form:

$$F(t) = \mathrm{Re}\{A_1 G_1(\omega_1) e^{i\omega_1 t} + A_2 G_1(\omega_2) e^{i\omega_2 t}\} + \frac{1}{2}\mathrm{Re}\{A_1^2 G_2(\omega_1,-\omega_1) + A_2^2 G_2(\omega_2,-\omega_2)\}$$

$$+\frac{1}{2}\mathrm{Re}\{A_1^2 G_2(\omega_1,\omega_1) e^{i2\omega_1 t}\} + \frac{1}{2}\mathrm{Re}\{A_2^2 G_2(\omega_2,\omega_2) e^{i2\omega_2 t}\}$$

$$+ \mathrm{Re}\{A_1 A_2 G_2(\omega_1,\omega_2) e^{i(\omega_1+\omega_2)t}\} + \mathrm{Re}\{A_1 A_2 G_2(\omega_1,-\omega_2) e^{i(\omega_1-\omega_2)t}\} \quad (20)$$

In equation (20) the first and second terms which are proportional to A_1 and A_2, respectively, are the linear exciting forces, which will be ignored in the present work. These are responsible for the motions of ships in waves as predicted by the standard seakeeping programs, but the motions are periodic, average out to zero, and thus are not of interest for the pure maneuvering problem.

The remaining terms are the nonlinear wave forces. These are quite small in magnitude relative to the first order forces, and are generally neglected in standard seakeeping studies. Several of these are important for stationkeeping and trackkeeping studies, because while they are small, their mean is non-zero and can cause considerable excursions in surge, sway, and yaw, where there are no restoring forces to balance them. The next two terms, proportional to A_1^2 and A_2^2, respectively, are the mean drift forces, and are important. The next three terms, proportional to $e^{i2\omega_1 t}$, $e^{i2\omega_2 t}$, and $e^{i(\omega_1+\omega_2)t}$, respectively, act at even higher frequencies than the incident waves, and thus are negligible. The last term, proportional to $A_1 A_2 e^{i(\omega_1-\omega_2)t}$, results in a slowly-varying drift force which is often also neglected, but may be important in trackkeeping with strict requirements on the amount of drift permitted from the nominal trackline. This is also important for stationkeeping, but is traditionally neglected in the static force balance approach often used in these problems. In the two wave case considered here, the slowly-varying force is a simple sinusoidal beat at the difference frequency. The two main expressions for forces of equation (20) can be extended to represent the responses to an irregular

seaway in terms of single and double integrals (summations for the discrete case). Most of the terms required in such an expression are unknown, however, and an approximate method must be found, utilizing only the mean drift force operator $G_2(\omega,-\omega)$.

Based on the discussion given above, only the slowly-varying drift forces will be included in this model. That is,

$$F_{WAVE} = F_{SVDF}$$

where F_{SVDF} will be defined in the next section utilizing the mean drift force operator $G_2(\omega,-\omega)$ which may be obtained from experimental data or from predictions.

Slowly-Varying Wave Drift Force Model

In this program, slowly-varying wave drift forces in short-crested waves are calculated by an approximate method due to Marthinsen [18]. This method is based on the assumption of a narrow-banded spectrum and the idea of a local wave frequency and direction to enable the use of regular wave drift force operators as the basis of the calculation. This is a more systematic and general approach which is equivalent to the approximation of Hsu and Blenkern[19]. In the case of a long-crested wave, Marthinsen showed through several examples that this approach is equivalent to the commonly used approximate methods of Newman[20]. This approximation uses only the regular wave drift force, which is the most readily available from experiment or calculation. These forces are calculated in the yawed system.

We will not derive the method here. The calculation method is as follows:
Calculate the Hilbert transform of the wave elevation, which is given by

$$\eta(x_0,y_0,t) = \mathrm{Im} \sum_{j=1}^{n} \sum_{k=1}^{m} e^{-iK_k(x_0 \cos\mu_j + y_0 \sin\mu_j) + i\omega_k t + i\varepsilon_{jk}} \sqrt{S_\zeta(\omega_k,\psi_j)\, \Delta\mu_j\, \Delta\omega_k}$$

for the short-crested case and by

$$\eta(x_0,y_0,t) = \mathrm{Im} \sum_{k=1}^{m} e^{-iK_k(x_0 \cos\mu_j + y_0 \sin\mu_j) + i\omega_k t + i\varepsilon_k} \sqrt{2S_\zeta(\omega_k)\, \Delta\omega_k}$$

for the long-crested case. Then the instantaneous wave amplitude, frequency, wave number components, and wave direction are given by:

$$A(t) = \sqrt{\zeta(t)^2 + \eta(t)^2}$$

$$\omega(t) = \left| \eta(t)\frac{\delta\zeta(t)}{\delta t} - \zeta(t)\frac{\delta\eta(t)}{\delta t} \right| / A^2(t)$$

$$k_x(t) = \left(\eta(t)\frac{\delta\zeta(t)}{\delta x} - \zeta(t)\frac{\delta\eta(t)}{\delta x} \right) / A^2(t)$$

$$k_y(t) = \left(\eta(t)\frac{\delta\zeta(t)}{\delta y} - \zeta(t)\frac{\delta\eta(t)}{\delta y} \right) / A^2(t)$$

$$\psi(t) = \tan^{-1}\left[\frac{k_y(t)}{k_x(t)} \right]$$

The resulting approximate expressions for the slowly-varying drift forces and moment are

$$X_{SVDF} = \frac{1}{2} \rho g L A^2(t) C_X(\omega(t), \psi(t))$$

$$Y_{SVDF} = \frac{1}{2} \rho g L A^2(t) C_Y(\omega(t), \psi(t))$$

$$N_{SVDF} = \frac{1}{2} \rho g L^2 A^2(t) C_N(\omega(t), \psi(t))$$

where in the previous notation

$$C_X(\omega(t), \psi(t)) = \frac{G_2 (\omega, -\omega)}{\frac{1}{2} \rho g L}$$

represents the non-dimensional mean surge drift force for a regular wave of frequency ω and direction ψ and C_Y and C_N are similarly the non-dimensional drift force and moment for sway and yaw, respectively.

In the simulation, the drift forces are updated only once per second and are assumed constant during the one second time interval. Further, in the initial phase of the simulation, the forces are ramped up from 0 at $t = 0$ to the full value at $t = t_{eramp}$, which is an input variable.

Strictly speaking, the slowly-varying drift force model is valid only for zero forward speed, because experimental data is available only at zero speed. Utilizing the model at low speeds is reasonable, however. At high speeds, drift forces are less significant because the hull forces which are proportional to speed squared are dominant and can easily compensate for the effect of drift forces.

TOW FORCES

The force, F_{TOW}, due to towing a body through the water is also modelled. It is assumed that the tow cable is parallel to the total ship velocity, U, and that the force is proportional to the square of the total ship velocity. The tow point is located at (x_T, y_T, z_T).

$$K_{TOW} = T_y z_T$$

$$M_{TOW} = T_x z_T$$

$$N_{TOW} = T_y x_T - T_x y_T$$

where

$$T_x = \cos\beta \, D_{TOW} \frac{U^2}{U_{TOW}^2},$$

$$T_y = \sin\beta \, D_{TOW} \frac{U^2}{U_{TOW}^2},$$

and D_{TOW} is the drag at a reference speed of U_{TOW}.

SOLUTION OF THE EQUATIONS OF MOTION

The program first initializes the state vectors for the ship and then the equations of motion are integrated forward in time to obtain the specific solution for a given set of initial conditions and rudder and speed commands. Turns, spirals, zig-zags, and arbitrary sequences of rudder and speed changes may be commanded.

The initial conditions for the time domain solution are chosen to represent the steady state *solution corresponding to the ship* moving at constant speed and heading, subject to the wind and waves assumed. Mean values of unsteady wind and slowly-varying wave drift forces are used during the initialization. The unsteady parts are ramped up from zero to full value at a (user-specified) time after the beginning of the run. This is necessary in order to prevent spurious starting transients in the solution.

The initial conditions are determined by minimizing the sum of the squares of the accelerations in six degrees of freedom. This yields rudder angle, shaft horsepower, heave, roll, pitch, and yaw (drift angle) to balance wind forces and forces due to asymmetry of the ship calm water hydrodynamic model.

The method adopted for finding initial conditions yields solutions which result in negligible starting transients. This has been demonstrated by running calm water solutions with no steering or speed commands for periods of time up to 1000 seconds (full scale) which resulted in essentially no change in the ship speed, heading, or attitude.

At any time t, the equations of motion, see equations (1-6), can be solved algebraically for the acceleration in terms of the velocities, displacements, and forcing functions. Force and moment components for hydrostatic, wind, wave drift, and tow forces are transformed to the maneuvering coordinate system using equation (13) and then added to the calm water hydrodynamic forces as in equation (14). Together with the kinematic relations of

equations (7-12) for the time derivatives of the position and orientation, and for certain variables used in the (optional) machinery models, these form a system of differential equations which are integrated numerically from the initial conditions as described in the previous section to find the motion of the ship as a function of time. In the present program, an adaptive Runge-Kutta method developed by Merson[21] is used.

Test runs in calm water conditions showed that predictions of sharp turns (rudder angle 35 degrees) were practically independent of step siz. for step sizes ranging from 0.0333 seconds to 5.0 seconds. For step sizes greater than 1.0 seconds, the step reduction feature came into play in the initial portion of the run when accelerations were largest.

CONCLUSION

This report describes a mathematical model to predict the motion of a surface ship while maneuvering in calm water, including unsteady wind and slowly-varying wave drift forces, and forces due to a towed body. This model incorporates a model of propeller-rudder interaction based on the physics of the situation, rather than a simple curve fit to experimental data as has been used previously. This mathematical model has been implemented in a computer program, MPSS (Maneuvering Program for Surface Ships). Hydrodynamic coefficients must be provided for the mathematical model. In addition, terms used in previous programs which are not included in the model here may optionally be included. This program is applicable to SWATH ships as well as monohulls.

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APPENDIX - ADDITIONAL TERMS IN WAVMAN44 OR TOWMAN87

The purpose of this appendix is to present terms which are included in the calm water forces and moments in WAVMAN44 or TOWMAN87, but are not included in the model presented in this report. An option exists in MPSS so that these additional terms can be utilized. This makes it possible to run MPSS using old data sets.

WAVMAN44 is a four-degrees-of-freedom program so that heave and pitch are not included. In WAVMAN44, U is defined as $\sqrt{u^2 + (w \cos \phi)^2}$, rather than $\sqrt{u^2 + v^2 + w^2}$. This U is used to dimensionalize the forces and moments. In addition, WAVMAN44 includes some of the terms which are multiplied by F in the model presented in this report. However in WAVMAN44 those terms are multiplied by $\bar{u}$. $\bar{u}$ is defined as $(u - U_0)$ and U_0 is the initial forward speed of the ship. When the WAVMAN44 option is utilized in MPSS, the definition of U stated here and the substitution of $\bar{u}$ for F will be implemented.

The additional terms in WAVMAN44, given as X_{CW}^{WM}, Y_{CW}^{WM}, K_{CW}^{WM} and N_{CW}^{WM} are:

$$X_{CW}^{WM} = X_v v + X_{r|v|} r |v| + X_{vvFF} v^2 F^2$$

$$+ X_{\bar{u}\delta\delta F} \bar{u} \delta^2 F + X_\delta \delta + X_{\delta\delta\bar{u}} \delta^2 \bar{u} + X_{\delta\bar{u}\bar{u}} \delta \bar{u}^2$$

$$+ X_{v\delta} v \delta + X_{r\delta} r \delta + X_{|\delta|v} |\delta| v + X_{\delta|v|} \delta |v| + X_{\delta vv} \delta v^2$$

$$Y_{CW}^{WM} = Y_0 + Y_{0\bar{u}} \bar{u} + Y_{vv} v^2 + Y_{vvv} v^3 + Y_{vrr} v r^2 + Y_{rrr} r^3$$

$$+ Y_{rvv} r v^2 + Y_{r\sqrt{|v|}} r \sqrt{|v|} + Y_{\phi|r|} \phi |r| + Y_{|\phi|r} |\phi| r + Y_{\phi\phi\phi} \phi^3 + Y_{\delta\phi} \delta \phi$$

$$+ Y_{vr\delta} v r \delta + Y_{\delta vv} \delta v^2 + Y_{\delta rr} \delta r^2 + Y_{\delta\delta} \delta^2 + Y_{v\delta\delta} v \delta^2 + Y_{r\delta\delta} r \delta^2 + Y_{\delta vr} \delta v r$$

$$+ Y_{\delta|v|} \delta |v| + Y_{v|\delta|} v |\delta| + Y_{\delta\bar{u}} \delta\bar{u} + Y_{\delta\delta\bar{u}} \delta^2 \bar{u} + Y_{\delta\delta\delta u} \delta^3 \bar{u}$$

$$K_{CW}^{WM} = K_{p\bar{u}} p \bar{u} + K_{p|p|} p |p| + K_{p|p|\bar{u}} p |p| \bar{u} + K_{rrr} r^3 + K_{rrv} r^2 v$$

$$+ K_{rvv} r v^2 + K_{vF} v F + K_{vFF} v F^2 + K_{p/F} p / F + K_{pF} p F + K_{pFF} p F^2$$

$$+ K_{|\phi|r} |\phi| r + K_{\phi|r|} \phi |r| + K_{\phi\phi\phi} \phi^3 + K_{\delta|v|} \delta |v| + K_{\delta\bar{u}} \delta\bar{u}$$

$$N_{CW}^{WM} = N_0 + N_{0\bar{u}} + N_{vv} v^2 + N_{vvv} v^3 + N_{vrr} v r^2$$

$$+ N_{rrr} r^3 + N_{rvv} r v^2 + N_r \sqrt{|v|}\sqrt{|v|} + N_{\phi|r|} \phi |r| + N_{|\phi|r} |\phi| r + N_{\phi\phi\phi} \phi^3$$

$$+ N_{\phi|\delta|} \phi |\delta| + N_{vr\delta} v r \delta + N_{\delta vv} \delta v^2 + N_{\delta rr} \delta r^2 + N_{\delta\delta} \delta^2$$

$$+ N_{v\delta\delta} v \delta^2 + N_{r\delta\delta} r \delta^2 + N_{\delta vr} \delta v r + N_{\delta|v|} \delta |v| + N_{v|\delta|} v |\delta| + N_{\delta\bar{u}} \delta \bar{u}$$

$$+ N_{\delta\delta\bar{u}} \delta^2 \bar{u} + N_{\delta\delta\delta\bar{u}} \delta^3 \bar{u}$$

While TOWMAN87 includes some heave and pitch calm water hydrodynamic and hydrostatic terms, it does not include the necessary inertial terms and transformations between coordinate systems required for a six degrees of freedom model.

TOWMAN87 includes calm water forces for heave and pitch. However, it does not include the inertial terms and coordinate transformations required for a consistent six degrees of freedom model. In TOWMAN87, U is defined as $\sqrt{u^2 + (w \cos \phi)^2}$, rather than $\sqrt{u^2 + v^2 + w^2}$. This U is used to dimensionalize the forces and moments. In addition, TOWMAN87 includes the terms which are multiplied by F in the model presented in this report. However in TOWMAN87 those terms are multiplied by powers of U. When the TOWMAN87 option is utilized in MPSS, the definition of U stated here and the substitution of U for F will be implemented when the flag indicating TOWMAN87 data is set in the input. The additional terms in TOWMAN87, given as X_{CW}^{TM}, Y_{CW}^{TM}, M_{CW}^{TM}, and N_{CW}^{TM} are:

$$X_{CW}^{TM} = X_\delta \delta + X_{\delta U} \delta U$$

$$Y_{CW}^{TM} = Y_{vvv} v^3 + Y_{v\phi} v \phi + Y_{r\phi} r \phi + Y_{\delta v} \delta v$$

$$K_{CW}^{TM} = K_{v\phi} v \phi + K_{r\phi} r \phi$$

$$M_{CW}^{TM} = M_\phi \phi$$

$$N_{CW}^{TM} = N_{rv} r v + N_{vvv} v^3 + N_{v\phi} v \phi + N_{r\phi} v \phi + N_{\delta v} \delta v$$

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